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A Multiplicative Human Development Index

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Abstract

This working paper provides an axiomatic foundation of a multiplicative human development index as an alternative to the standard additive one. There are three key advantages that derive from our multiplicative formulation: one, that it is a theoretically well founded measure; two, that it does not impose the restriction of having a constant rate of substitution between health, education, and income; and three, that it allows the introduction of distributive considerations.
We also present a comparative analysis between the additive and the multiplicative indices, using the 2006 data corresponding to 125 countries. The empirical study illustrates well how our perception of human development depends on the aggregation formula.

Key words

Multiplicative index, axiomatization.

Resumen

En este documento de trabajo se presenta una axiomatización de un índice de desarrollo humano de carácter multiplicativo, como alternativa al indicador convencional. Esta nueva forma de medir el desarrollo humano presenta tres ventajas importantes: 1) resulta una medida bien fundamentada teóricamente; 2) no impone la restricción de una tasa de sustitución constante entre las variables que miden salud, educación y renta; y 3) permite la introducción de consideraciones distributivas en la medición.
El trabajo incluye un análisis comparativo del índice de desarrollo humano multiplicativo y el convencional, usando datos de 2006 para 125 países. Se observa que la percepción del grado de desarrollo depende de forma sustantiva del tipo de indicador elegido.

Palabras clave

Índice de desarrollo multiplicativo, axiomatización.

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CONTENTS

1. Introduction

THE United Nations proposed, in 1990, a protocol to measure countries' degree of development, based on Amartya Sen's idea of *functionings and capabilities* (Sen 1985). This protocol first identifies health, education and material wellbeing as the key human functionings. Then, it produces a number of indicators, the best known of which is the *(Additive) Human Development Index* (AHDI). This index consists of a simple average of the normalized values of three variables that approximate the social achievements in those functionings. Achievements in health are associated with the *life expectancy at birth* variable. Achievements in education are approximated by a mixture of two variables, these being *literacy rate* and a combined *gross enrollment rate* (with weights of 2/3 and 1/3 respectively). Finally, achievements in material wellbeing are measured through the log of standard GDP per capita. All of those variables are normalized to take values in the interval [0, 1] and then aggregated into an index given by the formula: $\text{AHDI} = \frac{1}{3}(a+b+c)$ where a, b, c, stand for the *normalized* values of those partial indices. The AHDI is to be interpreted as an index that ranks different countries according to their achievements in those three functionings (see for instance the last report, HDR06).

Several criticisms to this indicator have already appeared in the literature. They refer to the number and nature of the selected functionings, the choice of the variables that measure those functionings, or the lack of theoretical justification of the aggregation formula, among others. The reader is referred to the works of Osberg (1985), Osberg and Sharpe (2002), Philipson and Soares (2001), Pinilla and Goerlich (2003), Becker, Philipson and Soares (2005), or Herrero, Soler and Villar (2005) for a critical appraisal. We consider here two additional sources of concern that make the AHDI a problematic measure. The first one refers to the additive nature of the index. The second one to the lack of consideration of distributive aspects. Let us briefly go over those two points.

The additive structure of the index entails two substantial drawbacks. One is the kind of trade-offs between functionings it admits, as this entails assuming full substitutability between all of them (linear indifference curves). That implies, for instance, that no matter how bad the state of health, it can always be compensated with further education or income *at a constant rate*. The second drawback derives from the sensitivity of the index with respect to the way the constituent variables are normalized. It is easy to verify that a change in the parameters chosen to normalize the variables *a, b, c*, may affect not only the resulting magnitudes of the AHDI, but also the ranking they produce. The reason is that, from an analytical standpoint, a change in the units in which partial indices are expressed cannot be distinguished from a change in the weights with which those variables enter the additive AHDI. So this index, with or without a theoretical justification, cannot be taken as a robust ordinal measure of development.

The absence of distributive considerations seems somehow inconsistent with the very nature of the exercise of measuring human development. This refers specially to the case of the income indicator (see Anand and Sen 2000 for a discussion). There is some empirical evidence pointing out that many poor countries suffer from a highly unequal distribution of opportunities. And also that richer countries differ significantly in the distribution of those opportunities. Ignoring distributive aspects implies, therefore, a bias that hides part of the existing differences between countries. Some modifications of the AHDI that include distributive considerations have been proposed by Anand and Sen (1994; 2000), Hicks (1997), and Foster et al (2003).

We provide in this paper an axiomatization of a *multiplicative human development index* (MHDI), as an application of the theory of multi-attribute decision-making developed by Bossert and Peters (2000). The three axioms that lead to the main result are quite intuitive. The first one, monotonicity, says that the index is an increasing function of its arguments. The second one, minimal lower boundedness, establishes that the index takes on its minimal value when health or education are at their minimum levels for all members of the society. And the third one, independence, mimics the independence axiom in the expected utility theory. It establishes that the ranking of two societies that have a common value for one of the variables does not vary if we change this common value in the two societies by the same amount.

The new index, so obtained, exhibits three main advantages with respect to the AHDI. First, it avoids the aforementioned shortcomings of the additive formula by adopting a multiplicative form. Second, it is a theoretically well founded aggregation index. Namely, the axiomatic approach allows clarification of the value judgements underlying the way of measuring human development. And third, it opens the possibility of introducing distributive considerations in the evaluation of human development.

We present a comparative analysis of the standard AHDI and the multiplicative version proposed here (including a multiplicative form scaled down by income inequality), for a large number of countries with different degrees of development (125 countries whose data appeared in the 2006 Human Development Report). This exercise illustrates well the impact of the aggregation procedure on our perception of human development. We observe that those indices rank differently many of the countries considered, even when they are relatively homogeneous. Moreover, the empirical distribution of the indicators is substantially diverse.

The paper is organized as follows: sections 2, 3 and 4 develop the theoretical model, while the empirical illustration is the subject of section 5. Some few final comments are gathered in section 6.

2. The Reference Model

CONSIDER a society $N=\{1,...,n\}$ made of n agents whose level of human development is to be measured in terms of three different dimensions that we will refer to as *health*, *education* and *income*. We shall not discuss here the precise content of those variables, but rather assume that there is a well-established criterion to get numerical measures of them (i.e. we take for granted that there is an agreement on the way the individual achievements in those three dimensions are to be measured). Let $\mathbf{h}=(h_1,...,h_n)$, $\mathbf{e}=(e_1,...,e_n)$, $\mathbf{y}=(y_1,...,y_n)$, denote the vectors that describe the values of those three variables for the population[1]. Let H, E, Y denote the corresponding spaces to which those variables belong, and we shall assume, for the sake of simplicity, that they are compact cubes in $\mathbb{R}^n$. That is, we can write $H=[h^0, h^*]^n$, $E=[e^0, e^*]^n$, and $Y=[y^0, y^*]^n$, where superscript 0 is the minimum admissible value of the corresponding variable and superscript $*$ the maximum admissible one. We assume $h^* > h^0$, $e^* > e^0$, $y^* > y^0$, to avoid trivialities. An n-vector whose all entries are equal to one is simply denoted by **1.**

Let $Q=H\times E\times Y$ denote the joint domain of those vector-variables. We define a **human development index** as a *continuous* function $D : Q \rightarrow \mathbb{R}$ such that the triple $(\mathbf{h}, \mathbf{e}, \mathbf{y})$ is considered at least as good as the triple $(\mathbf{h}', \mathbf{e}', \mathbf{y}')$, from the human development viewpoint, if and only if $D(\mathbf{h}, \mathbf{e}, \mathbf{y}) \geq D(\mathbf{h}', \mathbf{e}', \mathbf{y}')$. We also assume from the start that D is a cardinal measure; that is, the two indices $D, D' : Q \rightarrow \mathbb{R}$ represent the same ordering of human development levels if and only if, for each $(\mathbf{h}, \mathbf{e}, \mathbf{y}) \in Q$ we have: $D'(\mathbf{h}, \mathbf{e}, \mathbf{y}) = \lambda D(\mathbf{h}, \mathbf{e}, \mathbf{y}) + \mu$, with $\lambda, \mu \in \mathbb{R}$ and $\lambda > 0$. Therefore, since D is assumed to be continuous and the domain is compact, we can take the mapping D defined from Q into the interval $[0, 1]$ without loss of generality. Let $\mathcal{D}$ denote the domain of those normalized indices.

Now we consider three properties that introduce some value judgements on the evaluation formula. The first one, *monotonicity*, establishes that the human development index D increases when all the arguments increase

1. Note that we assume an individual measurement of those variables, rather than the usual population's average.

simultaneously. Note that this axiom implies that $D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{1}y^*) \geq D(\mathbf{h, e, y}) \geq D(\mathbf{1}h^0, \mathbf{1}e^0, \mathbf{1}y^0)$, for all $(\mathbf{h, e, y}) \in Q$ with $D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{1}y^*) > D(\mathbf{1}h^0, \mathbf{1}e^0, \mathbf{1}y^0)$.

Formally:

Monotonicity. For each, $(\mathbf{h, e, y}), (\mathbf{h', e', y'}) \in Q$, with $\mathbf{h} >> \mathbf{h}'$, $\mathbf{e} >> \mathbf{e}'$, and $\mathbf{y} >> \mathbf{y}'$, we have: $D(\mathbf{h, e, y}) > D(\mathbf{h', e', y'})$, $D(\mathbf{h}, \mathbf{1}e^*, \mathbf{1}y^*) > D(\mathbf{h}', \mathbf{1e}^*, \mathbf{1}y^*)$, $D(\mathbf{1}h^*, \mathbf{e}, \mathbf{1}y^*) > D(\mathbf{1}h^*, \mathbf{e}', \mathbf{1}y^*)$ and $D(\mathbf{1}h^*, \mathbf{1}\boldsymbol{e}^*, h^*) > D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{y}')$.[2]

The other two properties refer to the trade-offs between the variables in particular circumstances (Bossert and Peters 2000). *Minimal lower boundedness* states that there is no trade-off between development aspects when all members of the society are at their worst levels either in health or in income. This captures the idea that the contribution of education to development requires health and income to be above a minimum threshold. This is obvious if we think of h^0 as death y^0 and as a zero future income. Yet we can think of less dramatic thresholds that represent minimal values below which no compensation between education and health or income is possible.

Formally:

Minimal lower boundedness. For each $(\mathbf{h, e, y}), (\mathbf{h', e', y'}) \in Q$, $D(\mathbf{h, e, y}) \geq D(\mathbf{1}h^0, \mathbf{e}', \mathbf{y}')$ and $D(\mathbf{h, e, y}) \geq D(\mathbf{h}', \mathbf{e}', \mathbf{1}y^0)$.

The next property, *independence*, reminds of the independence axiom in the expected utility theory (hence the name). It involves two complementary requirements. First, that if $(\mathbf{h, e, y})$ is considered at least as good as $(\mathbf{h', e', y})$ for a common value of $\mathbf{y}$ then this relation holds for all common values of $\mathbf{y}$. So when comparing two situations, the ranking produced by the health and education variables is not altered by (i.e, is independent of) any common income vector. Second, that the same is true with respect to the education variable, with respect to income and health.

Formally:[3]

Independence. Let $\mathbf{h}, \mathbf{h}' \in H$, with $\mathbf{h}, \mathbf{h}' >> \mathbf{1}h^0$, $\mathbf{e}, \mathbf{e}' \in E$, and $\mathbf{y}, \mathbf{y}' \in Y$, with $\mathbf{y}, \mathbf{y}' >> \mathbf{1}y^0$. Then: (a) If $D(\mathbf{h, e, y}) \geq D(\mathbf{h', e', y})$ then $D(\mathbf{h, e, y'}) \geq D(\mathbf{h', e', y'})$. And (b) if $D(\mathbf{h, e, y}) \geq D(\mathbf{h', e, y'})$ then $D(\mathbf{h, e', y}) \geq D(\mathbf{h', e', y'})$.

Our main result establishes that those three properties determine a multiplicative structure of the human development index. More precisely, the index is the product of three different functions, each of which depends on a specific vector-variable (health, education, and income). The proof is relegated to Appendix (on page 21).

2. In the frontier, the property is applied with respect to the variable not being at the top level.

3 We exclude the extreme lower values of income and health in order to avoid undesirable cross effects between this axiom and that of minimal lower boundedness.

Theorem 1 *An index in $\mathcal{D}$ satisfies monotonicity, minimal lower boundedness, and independence if and only if*

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = f(\mathbf{h}) \cdot g(\mathbf{e}) \cdot s(\mathbf{y}), \tag{1}$$

where, $f: H \to [0,1]$, $g: E \to [0,1]$, *and* $s: \to [0,1]$ *are monotonic and positive functions such that* $f(\mathbf{1}h^0) = s(\mathbf{1}y^0) = 0$ and $f(\mathbf{1}h^*) = g(\mathbf{1}e^*) = s(\mathbf{1}y^*) = 1$.

Theorem 1 establishes that assuming monotonicity, minimal lower boundedness and independence together amounts to choosing an evaluation function that is multiplicatively decomposable in partial indices *f, g* and *s*, for health, education and income, respectively. This result can be considered as the starting point to then define particular human development measures by introducing further requirements on the evaluation function. Those requirements may refer to additional operational properties and/or value judgements, and can be easily understood in terms of their implications on the partial indices *f, g* and *s*.

Note that this formula is sensitive to the distribution of the values of the partial indices. Namely, for any two vectors $(\mathbf{h}, \mathbf{e}, \mathbf{y}), (\mathbf{h}', \mathbf{e}', \mathbf{y}') \in Q$ such that $f(\mathbf{h}) + g(\mathbf{e}) + s(\mathbf{y}) = f(\mathbf{h}') + g(\mathbf{e}') + s(\mathbf{y}')$ function *D* decreases with the variance of the vector $[f(.), g(.), s(.)]$, with an obvious maximum for the case in which $f(.) = g(.) = s(.)$. This feature has an interesting interpretation when we use relative values for the different variables: the development evaluation is sensitive not only to the levels of the constituent variables, but also to the dispersion of the partial indices they generate (a substantial departure from the purely additive formulation). Therefore, to occupy a top position in the ranking, a country has to exhibit both a high level of health, education and income, and a balanced mix of all those three aspects.

3. A Closed Evaluation Formula. The Counterpart of the Standard AHDI

THE following property introduces a scale in the human development index by determining its value in three special cases. It says that when all agents achieve the maximum values of two of the three variables and, furthermore, have a common value for the third one, then the index takes on the normalized value of that common parameter.

Formally:[4]

Uniformity. $D(\mathbf{1}\alpha, \mathbf{1}e^*, \mathbf{1}y^*) = \dfrac{\alpha - h^0}{h^* - h^0}$, $D(\mathbf{1}h^*, \mathbf{1}\alpha, \mathbf{1}y^*) = \dfrac{\alpha - e^0}{e^* - e^0}$, and $D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{1}\alpha) = \dfrac{\alpha - y^0}{y^* - y^0}$

It is easy to verify that the uniformity axiom, when combined with monotonicity, minimal lower boundedness and independence, implies that:

$$D(\mathbf{1}\mu(h), \mathbf{1}\mu(e), \mathbf{1}\mu(y)) = \hat{\mu}(\mathbf{h}) \cdot \hat{\mu}(\mathbf{e}) \cdot \hat{\mu}(\mathbf{y}) \qquad (2)$$

where $\mu(\mathbf{h})$, $\mu(\mathbf{e})$, $\mu(\mathbf{y})$ denote the average values of the variables health, education and income, respectively. And $\hat{\mu}(\mathbf{h}) = \dfrac{\mu(\mathbf{h}) - h^0}{h^* - h^0}$, $\hat{\mu}(\mathbf{e}) = \dfrac{\mu(\mathbf{e}) - h^0}{e^* - e^0}$ $\hat{\mu}(\mathbf{y}) = \dfrac{\mu(\mathbf{y}) - y^0}{y^* - y^0}$ are the corresponding normalized values. That is, the product of the normalized means is the right human development index when there is perfect equality. This suggests that the right-hand side of Equation (2) can be taken as a human development index, provided we are not concerned with the distribution of the variables describing health,

4. Note that we introduce the normalized value of the means in order to keep the domain of the function.

education and income within the population, as implicitly assumed in the AHDI proposed by the United Nations.

This idea can be formalized by means of the following property, *mean equivalence*, that says that the society's relevant information is captured by the mean values of the three aforementioned variables.

Formally:

Mean equivalence. For a given function $k : \mathbb{R}^n \to \mathbb{R}$, and each $\mathbf{x} \in \mathbb{R}^n$ we have: $k(\mathbf{x}) = k(\mathbf{1}\mu(\mathbf{x}))$.

When we apply the properties of uniformity and mean equivalence to all three component functions, the result in Theorem 1 yields the following formula:

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = \frac{\mu(\mathbf{h}) - h^0}{h^* - h^0} \cdot \frac{\mu(\mathbf{e}) - e^0}{e^* - e^0} \cdot \frac{\mu(\mathbf{y}) - y^0}{y^* - y^0} \qquad (3)$$

Therefore, we define the **multiplicative human development index** as MHDI $(\mathbf{h}, \mathbf{e}, \mathbf{y}) = \hat{\mu}(\mathbf{h}) \cdot \hat{\mu}(\mathbf{e}) \cdot \hat{\mu}(\mathbf{y})$. This formula corresponds, precisely, to the multiplicative counterpart of the additive human development index AHDI $(\mathbf{h}, \mathbf{e}, \mathbf{y}) = \frac{1}{3}\left(\hat{\mu}(\mathbf{h}) + \hat{\mu}(\mathbf{e}) + \hat{\mu}(\mathbf{y})\right)$.

Note that assuming uniformity but not mean equivalence gives us plenty of space to introduce distributive considerations into the index. We do so in the next section focusing on the income variable.

4. Income Inequality and Human Development

THE mean equivalence property is not an appealing assumption because it disregards distributive considerations. We propose now a modification of this property with respect to the income component of the formula that avoids this shortcoming. Note that the concern for equality in income distribution is a meaningful value judgement, whereas caring about equality with respect to health and education makes sense or not depending on the way those variables are measured [see the discussion in Anand and Sen (1993) and the approaches in Hicks (1997) and Moreno-Ternero and Villar (2006)]. Be that as it may, the approach concerning the income component can be readily extended to the other two welfare aspects.

A standard way of introducing distributive considerations is by means of the notion of *egalitarian equivalent income.* For a given income distribution vector $\mathbf{y} = (y_1,...,y_n)$, and a given income evaluation function s the egalitarian equivalent income, $y^e(\mathbf{y})$, is defined as the amount of income that, when uniformly distributed among the agents, would make the society as well-off in development as with the actual distribution. That is, $y^e(\mathbf{y})$ is defined by the following equation: $s(\mathbf{1}y^e(\mathbf{y})) = s(\mathbf{y})$. Alternatively, one can write $y^e(\mathbf{y}) = \mu(\mathbf{y})[1-I(\mathbf{y})]$ where $I(.)$ is an inequality index related to the social evaluation function.[5] Moreover, the term $\mu(\mathbf{y})I(\mathbf{y})$ gives us the average loss that is due to the unequal distribution of income in this society. Conversely, for a given inequality index $I(.)$, it is possible to define an income evaluation function s.

5. This comes from defining an inequality index as follows: $I(.)=1-\frac{y^e(\mathbf{y})}{\mu(\mathbf{y})}$. When the implicit social evaluation function is symmetric and concave (i.e. anonymous and egalitarian), $\mu(\mathbf{y}) \leq y^e(\mathbf{y})$ (where equality holds only for the uniform distribution).

We propose to substitute in Equation (3) the mean $\mu(\mathbf{y})$ by the egalitarian equivalent income $y^e(\mathbf{y})$ through the index $I(.)$. If we do so, once monotonicity, minimal lower boundedness, and independence are required, we end up with the following formula:

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = \left(\frac{\mu(\mathbf{h}) - h^0}{h^* - h^0}\right) \cdot \left(\frac{\mu(\mathbf{e}) - e^0}{e^* - e^0}\right) \cdot \left(\frac{\mu(\mathbf{y})(1 - I(\mathbf{y})) - y^0}{y^* - y^0}\right)$$

that obviously depends on the chosen inequality index.

Note that the monotonicity of $s(.)$ imposes clear bounds on the class of inequality indices that are admissible. It is easy to check that the Gini index (indeed the whole family of generalized Gini indices) is compatible with the monotonicity property, whereas the best known in the entropy family are not.

Remark 1 *If we take the Gini coefficient as the appropriate inequality measure, we obtain*:

$$s(\mathbf{y}) = \frac{\mu(\mathbf{y})(1 - G(\mathbf{y})) - y^0}{y^* - y^0}$$

that coincides with the income evaluation formula introduced by Sen (1976) in his personalized goods approach *and also proposed by Anand and Sen (1994). Therefore, once distributive requirements are introduced through the Gini index, we end up with the formula:*

$$\mathrm{MHDI}_G(\mathbf{h}, \mathbf{e}, \mathbf{y}) = \hat{\mu}(\mathbf{h}) \cdot \hat{\mu}(\mathbf{e}) \cdot \left(\frac{\mu(\mathbf{y})(1 - G(\mathbf{y})) - y^0}{y^* - y^0}\right) \tag{4}$$

5. A Case Study

WE devote this section to illustrate the implications and differences between our multiplicative human development indices (with and without distributive considerations, MHDI and $MHDI_G$) and the additive human development index proposed by the United Nations, AHDI. We follow the methodology of the Human Development Report 2006 (HDR06) and use the corresponding data included in that report in order to measure health, education, income and Gini coefficient.[6] Among the 177 countries for which the United Nations computed the additive human development index, here we only consider 125 countries, since the Gini index is available in the HDR06 only for those.

The health variable is measured through the life expectancy at birth. According to HDR06, $h^0 = 25$ and $h^* = 85$. Therefore, for each country,

$$\hat{\mu}(\mathbf{h}) = \left(\frac{\mu(\mathbf{h}) - h^0}{h^* - h^0}\right) = \frac{\text{life expectancy in birth} - 25}{85 - 25}$$

The education variable is measured through a combination of the adult literacy rate and the gross enrollment ratio, with weights of 2/3 and 1/3, respectively. According to HDR06, $e^0 = 0$ and $e^* = 100$. Therefore, for each country,

$$\hat{\mu}(\mathbf{e}) = \left(\frac{\mu(\mathbf{e}) - e^0}{e^* - e^0}\right) = \frac{\left(\frac{2}{3}\text{adult literacy rate} + \frac{1}{3}\text{gross enrollment ratio}\right) - 0}{100 - 0}$$

$$= \frac{2}{3}\frac{\text{adult literacy rate} - 0}{100 - 0} + \frac{1}{3}\frac{\text{gross enrollment ratio} - 0}{100 - 0}$$

The income variable is measured through the GDP per capita. According to HDR06, $y^0 = 100$ and $y^* = 40{,}000$. Therefore, for each country,[7]

$$\hat{\mu}(\mathbf{y}) = \left(\frac{\mu(\mathbf{y}) - y^0}{y^* - y^0}\right) = \frac{\text{GDP per capita} - 100}{40{,}000 - 100}$$

6. We do not discuss here the nature of the variables used to measure health, education and income, which we find arguable in many respects.

7. The HDR06 proposes to measure income through a normalization of the logarithmic GDP per capita. In order to calculate the multiplicative human development index we keep the same normalization but we do not take logs.

Finally, the inequality coefficient is measured through the Gini index in Table 15 of the HDR06 (pages 335-338).

Table 1 shows the data corresponding to all those normalized means as well as three different human development indices. The first one is the standard human development index, as defined by the United Nations, AHDI. The second one is the multiplicative human development index when there is no concern for inequality, MHDI (Equation 3). Finally, the third one is the multiplicative human development index when income inequality is measured by the Gini coefficient, $MHDI_G$ (Equation 4). Notice that not all the countries in the HDR06 are listed here. We have selected a subset of them, for illustrative purposes. As in the United Nations' reports, countries are grouped in three different classes, depending on their human development level, and ordered according to the AHDI.

In Figure 1, we plotted on the horizontal axis the 125 countries in a decreasing order according to the AHDI. The vertical axis is the absolute value of the three indices considered in this work. Peaks in the red and green lines correspond to changes in the order of the MHDI and $MHDI_G$ indices with respect to AHDI. With this figure, we then show that the use of a multiplicative index alters the ranking of the countries. That is, they actually are not ordinally equivalent. There are two reasons that explain those differences between the multiplicative and the additive human development indices. First, the introduction of distributive considerations when considering the $MHDI_G$. And second, the negative effect of differences between the partial indices for a given value of the sum, when considering the MHDI (i.e., for a constant value of the sum, the product becomes larger the closer those partial indices are).

FIGURE 1: Human development indices, ranked by AHDI

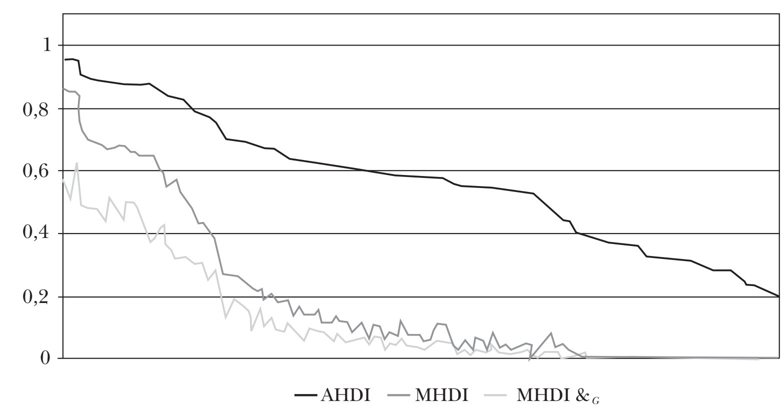


TABLE 1: Normalized means and human development indices for a collection of representative countries

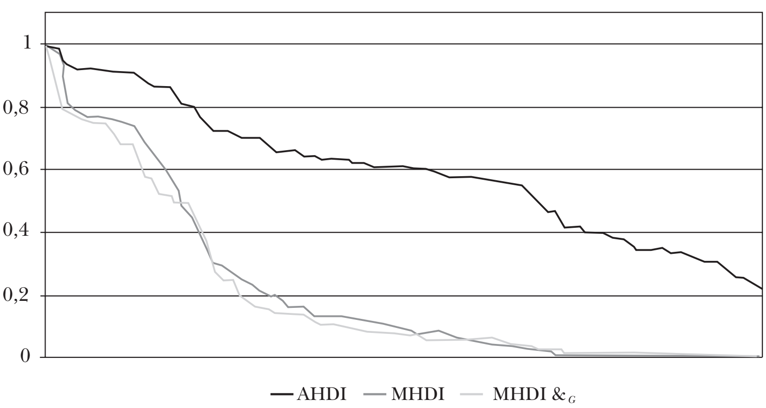
Country	Normalized means				Human Development Indices		
	$\hat{\mu}(\mathbf{h})$	$\hat{\mu}(\mathbf{e})$	$\hat{\mu}(\mathbf{y})$	$G(\mathbf{y})$	AHDI	MHDI	$MHDI_G$
High Human Development							
Ireland	0.88	0.99	1.00	0.34	0.957	0.871	0.572
United States	0.88	0.97	1.00	0.41	0.950	0.854	0.505
Canada	0.92	0.97	0.79	0.33	0.892	0.702	0.473
Belgium	0.9	0.98	0.79	0.33	0.889	0.694	0.465
Netherlands	0.89	0.99	0.79	0.31	0.889	0.693	0.479
Austria	0.9	0.96	0.79	0.29	0.882	0.679	0.482
United Kingdom	0.89	0.97	0.79	0.36	0.882	0.679	0.434
Denmark	0.87	0.99	0.79	0.25	0.882	0.677	0.510
Sweden	0.92	0.98	0.74	0.25	0.880	0.668	0.501
Japan	0.95	0.94	0.74	0.25	0.877	0.661	0.497
France	0.91	0.97	0.74	0.33	0.873	0.654	0.440
Finland	0.89	0.99	0.74	0.27	0.873	0.652	0.477
Italy	0.92	0.96	0.70	0.36	0.859	0.616	0.394
Germany	0.9	0.96	0.70	0.28	0.852	0.602	0.432
Spain	0.91	0.98	0.62	0.35	0.836	0.551	0.360
Greece	0.89	0.97	0.55	0.34	0.803	0.473	0.311
Portugal	0.87	0.96	0.49	0.39	0.772	0.406	0.250
Medium Human Development							
Brazil	0.76	0.88	0.21	0.58	0.616	0.140	0.059
Thailand	0.75	0.86	0.20	0.42	0.602	0.127	0.073
Colombia	0.79	0.86	0.18	0.59	0.612	0.126	0.052
Turkey	0.73	0.81	0.20	0.44	0.579	0.116	0.065
Iran	0.76	0.75	0.18	0.43	0.565	0.105	0.060
China	0.78	0.84	0.14	0.45	0.588	0.095	0.052
Peru	0.75	0.87	0.14	0.55	0.585	0.089	0.040
South Africa	0.37	0.8	0.28	0.58	0.484	0.084	0.035
Paraguay	0.77	0.86	0.12	0.58	0.584	0.080	0.034
Philippines	0.76	0.89	0.11	0.46	0.588	0.077	0.041
Ecuador	0.82	0.86	0.09	0.44	0.591	0.067	0.037
Egypt	0.75	0.73	0.10	0.34	0.527	0.055	0.036
Namibia	0.37	0.79	0.18	0.74	0.448	0.054	0.014
Morocco	0.75	0.54	0.11	0.40	0.466	0.043	0.026
Honduras	0.72	0.77	0.07	0.54	0.520	0.038	0.018
Bolivia	0.66	0.87	0.07	0.60	0.532	0.037	0.015
India	0.64	0.61	0.08	0.33	0.443	0.031	0.021
Mauritania	0.47	0.49	0.04	0.39	0.335	0.010	0.006
Guinea	0.48	0.34	0.05	0.40	0.290	0.008	0.005
Senegal	0.52	0.39	0.04	0.41	0.316	0.008	0.005
Low Human Development							
Kenya	0.37	0.69	0.03	0.43	0.362	0.007	0.004
Côte d'Ivoire	0.35	0.46	0.04	0.45	0.282	0.006	0.003
Rwanda	0.32	0.61	0.03	0.29	0.320	0.006	0.004
Nigeria	0.31	0.63	0.03	0.44	0.322	0.005	0.003
Benin	0.49	0.4	0.03	0.37	0.305	0.005	0.003
Mozambique	0.28	0.47	0.03	0.40	0.260	0.004	0.002
Tanzania	0.35	0.62	0.01	0.35	0.328	0.003	0.002
Zambia	0.21	0.63	0.02	0.42	0.287	0.003	0.002
Ethiopia	0.38	0.4	0.02	0.30	0.266	0.003	0.002
Central African Republic	0.24	0.42	0.03	0.61	0.228	0.003	0.001
Burundi	0.32	0.52	0.01	0.42	0.285	0.002	0.001
Burkina Faso	0.38	0.23	0.03	0.40	0.212	0.002	0.001
Malawi	0.25	0.64	0.01	0.50	0.301	0.002	0.001
Niger	0.33	0.26	0.02	0.51	0.202	0.001	0.001

We also observe here that, in absolute terms, the AHDI index dominates the MHDI index, and the latter does so with the $MHDI_G$ index. Finally, observe that the most noticeable changes in the ranking, once distributive aspects are introduced, refer to the most developed countries. Such changes are not so important if we consider the multiplicative human development index disregarding distributive aspects, MHDI.

The cardinal information that those indices provide is also substantially different. A first approximation comes from the analysis of their dispersion by some free scale measure. The *coefficient of variation* is a standard measure of this type. As one would expect, the dispersion is much larger for the $MHDI_G$ and the MHDI than for the AHDI. The coefficient of variation of the AHDI is 0.35, whereas the value for the MHDI is 1.27 (3.4 times that of the AHDI) and that for the $MHDI_G$ is 1.34 (3.8 times that of the AHDI). This suggests that the AHDI is hiding substantial differences between countries.

The comparative distribution of those three indices is presented in Figure 2, where we plotted on the horizontal axis the countries in a decreasing order according to the AHDI. In order to perform this comparison we have normalized each of the three human development indices by dividing each particular value by its corresponding maximum. In that way we obtain a measure that expresses the index as the percentage of the maximum achievement.

FIGURE 2: Relative human development indices, ranked by AHDI



6. Final Comments

WE have presented in this paper a theoretically well founded multiplicative human development index that exhibits some advantages with respect to the standard (additive) HDI. Most particularly, it allows for the introduction of distributive considerations into the setting and avoids constant rates of substitution between the variables. Those changes modify the assessment of the degree of development in actual societies, as Figures 1 and 2 show.

Several refinements are still possible. One of them has to do with the choice of the variables that measure health and education. In particular, life expectancy at birth may not be an adequate indicator of potential development, as it is independent of the population structure and thus tend to favor those countries with an older population. Moreover, the variable that measures education is not very informative when we consider developed countries. Another refinement deals with the extension of the distributive considerations in two different directions. First, by applying those considerations to health and education as well. Second, by using the equality of opportunity approach (Roemer 1998; Villar 2005) to separate those effects that depend on people's choices from those that can be attributed to people's external circumstances.

Appendix: Proof of the Theorem

The main result of the paper is the following:

Theorem 1. An index in $\mathcal{D}$ satisfies monotonicity, minimal lower boundedness, and independence if and only if

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{y})$$

where $f: H\to[0,1]$, $g: E\to[0,1]$, and $s: Y\to[0,1]$ are monotonic and positive functions such that $f(\mathbf{1}h^0) = s(\mathbf{1}y^0) = 0$ and $f(\mathbf{1}h^*) = g(\mathbf{1}e^*) = s(\mathbf{1}y^*) = 1$.
Proof: We present the proof in two steps. First we show that the multiplicative form satisfies the axioms. Then, that it is the only function form that does so.
(i) ($\Leftarrow$) We check the three axioms one by one.

Monotonicity. Let $(\mathbf{h}, \mathbf{e}, \mathbf{y}), (\mathbf{h}', \mathbf{e}', \mathbf{y}')\in Q$ such that $\mathbf{h} >> \mathbf{h}'$, $\mathbf{e} >> \mathbf{e}'$, and $\mathbf{y} >> \mathbf{y}'$. We know that f, g, s are monotonic, which implies that $f(\mathbf{h})> f(\mathbf{h}')$, $g(\mathbf{e})> g(\mathbf{e}')$, and $s(\mathbf{y})> s(\mathbf{y}')$. Since f, g, s are positive, we conclude that $D(\mathbf{h}', \mathbf{e}', \mathbf{y}')= f(\mathbf{h}')\cdot g(\mathbf{e}')\cdot s(\mathbf{y}') > f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{y}) = D(h, e, \mathbf{y})$. Similar arguments are applied to the frontier.

Minimal lower boundedness. Let $\mathbf{h}, \mathbf{h}'\in H$, $\mathbf{e}, \mathbf{e}'\in E$, and $\mathbf{y}, \mathbf{y}'\in Y$. Then $D(h, e, \mathbf{y}) = f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{y}) \geq f(\mathbf{1}h^0)\cdot g(\mathbf{e}')\cdot s(\mathbf{y}') = D(\mathbf{1}h^0, \mathbf{e}', \mathbf{y}')$ and $D(h, e, \mathbf{y}) = f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{y}) \geq f(\mathbf{h})\cdot g(\mathbf{e}')\cdot s(\mathbf{1}y^0) = D(\mathbf{h}', \mathbf{e}', \mathbf{1}y^0)$.

Independence. Let $\mathbf{h}, \mathbf{h}'\in H,$ with $\mathbf{h}, \mathbf{h}' >> \mathbf{1}h^0$, $\mathbf{e}, \mathbf{e}'\in [e^0, e^*]^n$ and $\mathbf{y}, \mathbf{y}'\in Y$ with $\mathbf{y}, \mathbf{y}'>> \mathbf{1}y^0$. Notice that monotonicity implies that $f(\mathbf{h}) > f(\mathbf{1}h^0) = 0$, and $s(\mathbf{y}) > s(\mathbf{1}y^0) = 0$.

(a) Suppose that $D(\mathbf{h}, e, \mathbf{y}) \geq D(\mathbf{h}', \mathbf{e}', \mathbf{y})$, then

$$\begin{aligned} D(\mathbf{h},\mathbf{e},\mathbf{y}) &\geq D(\mathbf{h}',\mathbf{e}',\mathbf{y}) \\ f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{y}) &\geq f(\mathbf{h}')\cdot g(\mathbf{e}')\cdot s(\mathbf{y}) \\ f(\mathbf{h})\cdot g(\mathbf{e}) &\geq f(\mathbf{h}')\cdot g(\mathbf{e}') \\ f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{y}') &\geq f(\mathbf{h}')\cdot g(\mathbf{e}')\cdot s(\mathbf{y}') \\ D(\mathbf{h},\mathbf{e},\mathbf{y}') &\geq D(\mathbf{h}',\mathbf{e}',\mathbf{y}') \end{aligned}$$

(b) Suppose that $D(\mathbf{h}, \mathbf{e}, \mathbf{1}) \geq D(\mathbf{h}, \mathbf{e}', \mathbf{1})$, then

$$\begin{aligned} D(\mathbf{h}, \mathbf{e}, \mathbf{1}y^*) &\geq D(\mathbf{h}, \mathbf{e}', \mathbf{1}y^*) \\ f(\mathbf{h})\cdot g(\mathbf{e})\cdot s(\mathbf{1}y^*) &\geq f(\mathbf{h})\cdot g(\mathbf{e}')\cdot s(\mathbf{1}y^*) \\ g(\mathbf{e})\cdot s(\mathbf{1}y^*) &\geq g(\mathbf{e}')\cdot s(\mathbf{1}y^*) \\ f(\mathbf{h}')\cdot g(\mathbf{e})\cdot s(\mathbf{1}y^*) &\geq f(\mathbf{h}')\cdot g(\mathbf{e}')\cdot s(\mathbf{1}y^*) \\ D(\mathbf{h}', \mathbf{e}, \mathbf{1}y^*) &\geq D(\mathbf{h}', \mathbf{e}', \mathbf{1}y^*) \end{aligned}$$

Therefore, any multiplicative index fulfils the three properties.

(ii) ($\Rightarrow$)

Let D be an index in $\mathcal{D}$ that satisfies *monotonicity*, *minimal lower boundedness*, and *independence*. By *monotonicity* and *minimal lower boundedness*

$$0 = D(\mathbf{1}h^0, \mathbf{1e}^0, \mathbf{1}y^0) \geq D(\mathbf{h}, \mathbf{e}, \mathbf{y}) \geq D(\mathbf{1}h^*, \mathbf{1e}^*, \mathbf{1}y^*) = 1 \text{ for each } (\mathbf{h}, \mathbf{e}, \mathbf{y}) \in Q$$

By *independence*, since $\mathbf{y}$ is independent of $\mathbf{h}$ and $\mathbf{e}$ for any pair $(\mathbf{h}, \mathbf{e}) \in H \times E$, there exist $u : H \times E \rightarrow \mathbb{R}_{++}$ and $v: H \times E \rightarrow \mathbb{R}$ such that:[8]

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = u(\mathbf{y}) + v(\mathbf{y})\ D(\mathbf{h}, \mathbf{e}, \mathbf{1}y^*) \tag{5}$$

By letting $\mathbf{h} = \mathbf{1}h^0$ in Equation (5), *minimal lower boundedness* implies that $u(\mathbf{y}) = 0$, for all $(\mathbf{h}, \mathbf{e}) \in H \times E$. Now, letting $\mathbf{h} = \mathbf{1}h^*$ and $\mathbf{e} = \mathbf{1}e^*$, we get

$$v(\mathbf{y}) = D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{y})$$

and, consequently,

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = D(\mathbf{1}h^*, \mathbf{1}e^*, y) \cdot D(\mathbf{h}, \mathbf{e}, \mathbf{1}y^*) \tag{6}$$

By *independence*, as $\mathbf{h}$ is independent of $\mathbf{e}$ and $\mathbf{y} = \mathbf{1}y^*$, for any $\mathbf{e} \in E$, there exist $u' : E \rightarrow \mathbb{R}_{++}$ and $v' : E \rightarrow \mathbb{R}$, such that

$$D(\mathbf{h}, \mathbf{e}, \mathbf{1y}^*) = u'(\mathbf{e}) + v'(\mathbf{e}) D(\mathbf{h}, \mathbf{1}e^*, \mathbf{1}y^*) \tag{7}$$

Now, letting $\mathbf{h} = \mathbf{1}h^0$ in Equation (7) and using again *minimal lower boundedness*, we get $u'(\mathbf{e}) = 0$ for all $\mathbf{e} \in E$. Additionally, letting $\mathbf{h} = \mathbf{1}h^*$, we get

$$v'(\mathbf{e}) = D(\mathbf{1}h^*, \mathbf{e}, \mathbf{1}y^*)$$

8. See Keeney and Raiffa (1975, chaps. 5, 6).

and, consequently,

$$D(\mathbf{h}, \mathbf{e}, \mathbf{1}y^*) = D(\mathbf{1}h^*, \mathbf{e}, \mathbf{1}y^*)D(\mathbf{h}, \mathbf{1}e^*, \mathbf{1}y^*) \quad (8)$$

Now, substituting (8) in (6), we conclude:

$$D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{y}) \cdot D(\mathbf{1}h^*, \mathbf{e}, \mathbf{1}y^*) \cdot D(\mathbf{h}, \mathbf{1}e^*, \mathbf{1}y^*)$$

If we define $f(\mathbf{h}) = D(\mathbf{h}, \mathbf{1}e^*, \mathbf{1}y^*)$, $g(\mathbf{e}) = D(\mathbf{1}h^*, \mathbf{e}, \mathbf{1}y^*)$, and $s(\mathbf{y}) = D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{y})$, we get the multiplicative form of the statement.

On the other hand, by *monotonicity* and *minimal lower boundedness*, $f(\mathbf{1}h^*) = g(\mathbf{1}e^*) = s(\mathbf{1}y^*) = D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{1}y^*) = 1$, $f(\mathbf{1}h^0) = D(\mathbf{1}h^0, \mathbf{1}e^*, \mathbf{1}y^*) = 0$, and $s(\mathbf{1}y^0) = D(\mathbf{1}h^*, \mathbf{1}e^*, \mathbf{0}) = 0$. Since $f(\mathbf{h})=D(\mathbf{h}, \mathbf{1}e^*, \mathbf{1}y^*)$ and D is *monotonic*, f is monotonic as well. Similar arguments show that g and s are monotonic. Besides, by *independence*, f and s are positive, and then g is as well.

Remark 2 *The* AHDI *satisfies monotonicity and independence but fails to satisfy minimal lower boundedness. A human development index that satisfies minimal lower boundedness and independence but does not satisfy monotonicity is, for instance,* $D(\mathbf{h}, \mathbf{e}, \mathbf{y}) = y^0$. *Finally, a human development index that satisfies monotonicity and minimal lower boundedness but does not satisfy independence is:* $D(\mathbf{h}, \mathbf{e}, \mathbf{y})=[(\mu(\mathbf{h})-h^0)+(\mu(\mathbf{e})-e^0)+(\mu(\mathbf{y})-y^0)]^2$. *The axioms that characterize the multiplicative index are, therefore, independent.*

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